

Neutrino Masses and $b - \tau$ unification in the Supersymmetric Standard Model

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ABSTRACT

There are several indications that the Majorana masses of the right-handed neutrino components, M_R , are at the intermediate scale: $M_R \sim (10^{10} - 10^{12})$ GeV or even lighter. The renormalization effects due to large Yukawa couplings of neutrinos from region of momenta $M_R \lesssim q \lesssim M_G$ are studied in the supersymmetric standard model. It is shown that neutrino renormalization effect can increase the m_b/m_τ ratio up to $(10 \div 15)\%$. This strongly disfavors $m_b - m_\tau$ unification for low values of $\tan \beta < 10$ especially at large values of α_s . Lower bounds on M_R and $\tan \beta$ from the $b - \tau$ unification condition were found. The implications of the results to the see-saw mechanism of the neutrino mass generation are discussed.

1. Introduction.

The unification of fermion masses, and in particular the masses of b -quark and τ -lepton [1] at the scale M_G , where the gauge coupling unify,

$$m_b(M_G) = m_\tau(M_G) \quad (1)$$

is one of crucial issues of the Grand Unification (GU in the following). With the standard model particle content neither $b - \tau$ unification nor gauge couplings unification happen to be realized [2]. In contrast, in the minimal supersymmetric extension of the standard model there is a successful gauge coupling unification [3]; also the $b - \tau$ unification is possible, but at the large Yukawa coupling of top (and/or of bottom) quark only, so that the renormalization effect due to the Yukawa interaction appreciably suppresses the value of m_b at low scale [4].

In previous studies it was suggested that the right-handed (RH) neutrino components (if they exist) are at the M_G or higher scale, so that the neutrinos do not contribute to the renormalization [5]. However, there are several indications that the Majorana masses of the RH neutrinos, M_R , are much smaller than the GU scale: $M_R \ll M_G$, and consequently the neutrino renormalization effects may be important. Indeed, studies of the solar, atmospheric, as well as relic (hot dark matter, large scale structure of the universe) neutrinos give some hints of the existence of nonzero neutrino masses [6]. The required values of masses can be naturally generated by the see-saw mechanism [7] if the Majorana masses of the RH neutrinos are at the intermediate scale: $M_R \sim (10^{10} - 10^{12})$ GeV. In particular, for the tau neutrino to be in the cosmologically interesting domain, $m_\nu \sim 3 - 10$ eV, one needs

$$M_R \lesssim m_t^2/m_\nu \sim 10^{11} \text{ GeV}. \quad (2)$$

The same scale of masses is required by mechanism of the baryon asymmetry generation based on the decay of the RH neutrinos [8]. Much lower masses: $M_R < 10^7$ GeV, are implied by the Primordial nucleosynthesis in the supersymmetric models with spontaneous violation of lepton number [9].

The RH neutrinos decouple at $q < M_R$, and if $M_R \ll M_G$, one should take into account the renormalization effect due to the Yukawa interactions of neutrinos from the region of momentum $M_R \lesssim q \lesssim M_G$. Moreover, in a wide class of the GU theories the Yukawa couplings of the neutrinos and up-quarks are of the same order (or even equal at the GU scale). Therefore, at least for the tau neutrino one can expect large Yukawa coupling, and consequently, large neutrino renormalization effect.

In this paper we study the renormalization effects of neutrinos on the ratio m_b/m_τ as well as on the neutrino masses in the supersymmetric model. As the first step the problem is worked out mainly in one loop approximation and the threshold effects of the new particles at GU scale as well as at electroweak scales are neglected.

2. Renormalization group equations with RH Neutrinos

Let us extend the minimal supersymmetric standard model by introducing the three right-handed neutrino superfields ν_i^c (one for each family), and adding to the (matter parity conserving) superpotential the terms with neutrino masses and the Yukawa interactions:

$$W = -QY_u^*U^cH_2 + QY_d^*D^cH_1 + LY_e^*E^cH_1 - LY_\nu^*\nu^cH_2 + \mu H_1H_2 + \frac{1}{2}\nu^cM\nu^c. \quad (3)$$

Here $Q, L, U^c, D^c, E^c, \nu^c$ are the matter superfields, H_1 and H_2 are the Higgs superfield doublets, and Y_i ($i = u, d, e, \nu$) are the 3×3 matrices of the Yukawa couplings. Taking into account the successful unification of gauge couplings in MSSM we will assume that there is no new particles apart from ν^c up to the Grand Unification scale. The right-handed neutrino do not influence the gauge coupling constants evolution at 1-loop level; the 2-loop effect is estimated to be small.

We will suggest first that the effect of flavour mixing is negligible, and there is a hierarchy of the Yukawa couplings, so that the renormalization effects of particles from the third family is important only. (We will comment on the effect of mixing in sect. 3).

It is convenient to write the renormalization group equation (RGE) for the couplings:

$$\alpha_x = \frac{|Y_x|^2}{4\pi} \quad (x = t, b, \tau, \nu_\tau), \quad (4)$$

being the analogous of the gauge coupling constants. In terms of α_x and the parameter $\tan \beta$, where $\tan \beta \equiv v_2/v_1$, and v_1 and v_2 are the vacuum expectation values of the Higgs doublets H_1, H_2 , the masses of the quarks and leptons at the Z^0 -mass scale, M_Z , can be written as:

$$\begin{aligned} m_{t,\nu_\tau}(M_Z) &= \sqrt{4\pi\alpha_{t,\nu_\tau}(M_Z)} \frac{\tan \beta}{\sqrt{1+\tan^2 \beta}} v \\ m_{b,\tau}(M_Z) &= \sqrt{4\pi\alpha_{b,\tau}(M_Z)} \frac{1}{\sqrt{1+\tan^2 \beta}} v. \end{aligned} \quad (5)$$

Here $v \equiv \sqrt{v_1^2 + v_2^2}$. Note that for fixed mass m_b the bottom coupling α_b increases with $\tan \beta$, whereas the top coupling increases when $\tan \beta$ diminishes. Evidently, $m_b/m_\tau = \sqrt{\alpha_b/\alpha_\tau}$.

Applying, for instance, the method of the effective potential of ref. [10], one finds

the RGE for the couplings α_x :

$$\alpha'_t = \left(\sum_i b_u^i \alpha_i - 6\alpha_t - \alpha_b - \alpha_{\nu_\tau} \theta_R \right) \alpha_t \quad (6)$$

$$\alpha'_b = \left(\sum_i b_d^i \alpha_i - 6\alpha_b - \alpha_t - \alpha_\tau \right) \alpha_b \quad (7)$$

$$\alpha'_{\nu_\tau} = \left(\sum_i b_\nu^i \alpha_i - 4\alpha_{\nu_\tau} \theta_R - \alpha_\tau - 3\alpha_t \right) \alpha_{\nu_\tau} \quad (8)$$

$$\alpha'_\tau = \left(\sum_i b_e^i \alpha_i - 4\alpha_\tau - \alpha_{\nu_\tau} \theta_R - 3\alpha_b \right) \alpha_\tau, \quad (9)$$

where $A' \equiv dA/dT$, $T = 1/2\pi \log(M_G/Q)$, and $\theta_R(T) \equiv \theta(T - T_R)$ is the step function which describes the effect of the ν_R decoupling at M_R ($T_R \equiv 1/2\pi \log(M_G/M_R)$). The gauge coupling constants α_i ($i = SU_3, SU_2, U_1$) enter, in particular, the neutrino RGE with the coefficients: $b_\nu = (0, 3, \frac{3}{5})$. The signs in the eqs. (6-9) reflect the well known fact that during the evolution from the high to the low energy scale, the effect of the gauge coupling constants is to increase the Yukawa couplings, while the effect of the Yukawa itself is opposite.

The following preliminary remarks are in order. Neglecting all the Yukawa coupling but α_t in (6) (which is justified for small values of $\tan \beta$) one gets the equation

$$\alpha'_t = \left(\sum_i b_u^i \alpha_i - 6\alpha_t \right) \alpha_t. \quad (10)$$

For large values of α_t its solution has the well known IR fixed point behaviour, which is kept when the system of equations is taken into account without simplification. At large scales α_t diverges; small variations of $\alpha_t(T_Z)$, and therefore $m_t(M_Z)$ result in strong changes of α_t at the GU scale (here $T_Z \equiv 1/2\pi \log(M_G/M_Z)$). It is worthwhile to notice that the running top mass $m_t(M_Z)$ coincides numerically with the pole mass M_t up to 2% if M_t is in the range 150 – 200 GeV.

In fig. 1a-c we show the divergency lines on the $m_t(M_Z) - \tan \beta$ -plot which correspond to $\alpha_t(0) = 0.5$ at M_G . They were found by numerical solution of the system (6-9) at maximally admitted value of m_b which corresponds to the pole mass $M_b^{pole} = 5.2$ GeV. For values of the parameters $m_t(M_Z) - \tan \beta$ above the divergency lines the perturbative approach is invalid, the mass m_t blows up before GU scale. The increase of gauge coupling α_s results in a decrease of the α_t at large scales, and consequently, relaxes the divergency bound on $m_t(M_Z)$ (fig. 1a-c). The Yukawa couplings have an opposite effect. This explains the bending of the curves for large values of $\tan \beta$ (according to (5) an increase of $\tan \beta$ corresponds to increase of the bottom coupling α_b). Neutrino renormalization also makes the divergency bound on m_t more stringent. The corrections due to neutrinos: $|\Delta m_t| \lesssim 5$ GeV, are comparable with the uncertainty related to α_s .

3. Renormalization of m_b/m_τ and the $b - \tau$ unification

We will suggest the $b - \tau$ unification, so that the b and τ Yukawa couplings coincide at M_G :

$$\alpha_b(0) = \alpha_\tau(0). \quad (11)$$

For definiteness we also suggest the equality of the couplings of the top quark and the tau neutrino:

$$\alpha_t(0) = \alpha_{\nu_\tau}(0). \quad (12)$$

In fact, the equalities (11, 12) appear in GU theories with the Higgs multiplets of the lower dimension*.

Using (6-9) one finds the RGE for the mass ratio:

$$\left(\frac{m_b}{m_\tau}\right)' = \frac{1}{2} \left(\sum_i (b_d^i - b_e^i) \alpha_i - 3(\alpha_b - \alpha_\tau) - (\alpha_t - \alpha_{\nu_\tau} \theta_R) \right) \left(\frac{m_b}{m_\tau}\right). \quad (13)$$

The coefficient $1/2$ reflects that $m \sim \sqrt{\alpha}$. Note that the effect of down quark and charge lepton is enhanced by factor 3. The following conclusions can be drawn from eq. (13) immediately. (i) The increase of α_t , or of α_b tend to decrease the ratio. This is a key ingredient to achieve the $b - \tau$ unification. (ii) The gauge coupling α_s works in opposite direction: the predicted value of the m_b/m_τ ratio increases with α_s . (iii) The neutrino renormalization also results in increase of m_b/m_τ . Therefore to achieve the mass unification one should take larger values of α_t to compensate the effects of neutrino or/and α_s .

These features can be seen in fig. 2 a-c. The ratio of masses of m_b/m_τ predicted from the $b - \tau$ unification is shown as function of $\alpha_t(0)$ at the GU scale for different values M_R and α_s . The prediction is compared with the experimental mass ratio which has been found by converting the pole masses in running masses, and running them from low energy scale to M_Z . (We take into account two-loop SM renormalization effects as explained in [2]). The two horizontal lines in fig. 2 correspond to two values of pole mass: $M_b^{pole}(\text{max}) = 4.85 \text{ GeV}$ (being 2σ above the value quoted in [11]) and $M_b^{pole}(\text{max}) = 5.2 \text{ GeV}$ (an extreme upper bound). As follows from the figure, the agreement between the predicted and the experimental (running) values can be achieved only for large values of $\alpha_t(0)$ and of $\tan\beta$ (*i.e.* large α_b). A decrease of M_R results in increase of the ratio. The neutrino renormalization effect prevents from the $b - \tau$ unification at small $\tan\beta$:

*There are no similar relations for 1^{st} and 2^{nd} generations. One can suggest that only the particles from the third generation acquire the masses at the tree level in interactions with Higgs multiplets. Masses of the lightest generations appear as radiative corrections or/and from effective nonrenormalizable terms.

$\tan \beta < 10$. If for example $M_R < 10^{-3} M_G$, no unification can be achieved for $\alpha_s > 0.115$ and α_t up to the divergency limit. The reliable upper bounds on m_t will allow to further strengthen the bounds on $\tan \beta$ and M_R with respect to those from the divergence of $\alpha_t(0)$. Indeed, taking as an example the case $\alpha_s(M_Z) = 0.12$ and $\tan \beta = 3$, one gets that the top mass range $m_t = 164 \div 184$ GeV [12] corresponds to $\alpha_t(0) = 0.027 \div 0.55$ for $M_R = 10^{-6} M_G$ and $\alpha_t(0) = 0.025 \div 0.22$ for $M_R = M_G$. Consequently, the upper bound on m_t can give an upper bound on $\alpha_t(0)$ which is appreciably lower than the divergence limit.

The dependence of the limits for $\tan \beta$ and M_R on m_t can be seen in fig. 3. Here the $b - \tau$ unification curves are shown in the $(m_t(M_Z) - \tan \beta)$ -plane. The curves were found by solving the system (6-9) and correspond to the highest possible value $M_b^{pole} = 5.2$ GeV. Evidently a situation for smaller b-masses is even worser. As follows from fig. 3 in case of the decoupled RH neutrinos, $M_R = M_G$, the unification solution exists in all the range of the $\tan \beta$ values. With a decrease of $\tan \beta$ (decrease α_b) the dump effect of the bottom Yukawa coupling on m_b/m_τ decreases. This should be compensated by the increase of the renormalization effect of the top quark which results in increase of the top mass. In the case $M_R \ll M_G$ the neutrino effects reduce the influence of the top interaction, and therefore for a given value of $\tan \beta$ the coupling $\alpha_t(0)$ has to be larger, in comparison with the case $M_R = M_G$. For this reason the unification curves of fig. 3 shift to larger values of m_t . Moreover, for small M_R with decrease of $\tan \beta$ these curves exit from the perturbative domain. The terminations of curves at some values of $\tan \beta$ correspond with a good precision to the divergency limit of fig. 1.

As follows from fig. 3 for light M_R the $b - \tau$ unification and the condition that α_t is in the perturbative domain allow one to get the lower bound on $\tan \beta$, and an upper bound on the top mass. For example, at $M_R = 10^{-6} M_G$ one finds $\tan \beta \gtrsim 31, 46, 55$ and $m_t \lesssim 188, 186, 184$ GeV for $\alpha_s = 0.115, 0.120$, and 0.125 correspondingly.

In fig. 4 we show explicitly the lower bounds on $\tan \beta$ and M_R obtained from the $b - \tau$ unification condition ($M_b^{pole} \leq 5.2$ GeV) and the convergency limit for α_t for different values of α_s . Let us stress a strong dependence of the bounds on strong coupling constant.

Evidently, for realistic values of M_b the conclusions derived from fig. 3 and 4 strengthen. Since the bottom Yukawa coupling is lower in this case, α_t has to be larger, and the unification curves closer to the divergency curves. This fact disfavors strongly the low $\tan \beta$ region also for low value of α_s .

Note that for relatively small values of $\tan \beta$, and consequently, the small Yukawa

coupling of b and τ , it is possible to write the explicit expression for the mass ratio. Neglecting the coupling constants α_b , α_τ in the brackets of (6-9), as well as α_{ν_τ} in (6) one finds the solution of the simplified system of equations:

$$\frac{m_b}{m_\tau}(T_Z) = \left(\frac{E_b(T_Z)}{E_\tau(T_Z)} \right)^{1/2} \frac{D_{\nu_\tau}(T_Z)^{1/8}}{D_t(T_Z)^{1/12}} \quad , \quad (14)$$

where the renormalization effect of the Yukawa couplings are summarized in the functions D_t and D_{ν_τ} , while the four functions E_x describe the gauge interaction effects:

$$\begin{aligned} E_x(T) &= \Pi_i \left(\frac{\alpha_i(T)}{\alpha_i(0)} \right)^{-b_x^i/b_i} & (x = t, b, \tau, \nu) \\ E_{\nu_\tau}(T) &= \frac{E_\nu(T)}{D_t(T)^{1/2}} \\ D_t(T) &= 1 + 6 \alpha_t(0) \int_0^T E_u(x) dx \\ D_{\nu_\tau}(T) &= 1 + 4 \alpha_{\nu_\tau}(0) \int_0^T E_{\nu_\tau}(x) \theta_R(x) dx \end{aligned} \quad (15)$$

The top and the neutrino contributions, that is $D_t(T)$ and $D_{\nu_\tau}(T)$, are related by eq. (12). The solution (14-15) generalizes the result of ref. [13] by including the neutrino effect.

For $\tan\beta < 10$ the approximate solution eq. (14) coincides with the results of the numerical integration of eqs. (6-9) within $2 \div 3\%$. Note that according to (15) at small $\tan\beta$ the Yukawa renormalization effects do not depend on m_t and on $\tan\beta$ separately: they enter the expression only via $\alpha_t(0)$. Consequently, for fixed $\alpha_t(0)$ the unification curves do not depend on $\tan\beta$. At $\tan\beta < 10$ the curves for different $\tan\beta$ practically coincide with those shown in fig. 2 for $\tan\beta = 3$.

4. Neutrino masses

Above the M_R scale eq. (8) gives us the RGE of the see-saw mass of the tau neutrino, being $m_{\nu_\tau} = k \alpha_{\nu_\tau}$ ($k = 4\pi v_2^2/M_R$, where M_R the physical mass of the RH tau neutrino). Also below this scale the renormalization of $k \alpha_{\nu_\tau}$ according to eq. (8) gives us the correct evolution of m_{ν_τ} since this evolution coincides with that of the mass operator, in the supersymmetric case, found recently in [14]. We will study in a future work the relation of this issue with the non renormalization theorems in supersymmetry. We include in the present analysis the effect of the large neutrino Yukawa coupling in the region of momenta above M_R ; this results in large renormalization effects for the neutrino masses themselves.

In the context of $b - \tau$ unification the value of $\alpha_t(0)$, and consequently $\alpha_{\nu_\tau}(0)$, are calculated once the top mass is fixed. Depending on value of M_R one or two solutions for $\tan\beta$ can exist (see fig. 3), and consequently there are one or two solutions for $\alpha_t(0)$. In

our calculation we take the solution with large values of $\tan\beta$, being interested to have lower values of M_R . For $m_t = 174 \pm 10$ GeV and $\alpha_s = 0.12$ the results of numerical solution of system (6-9) can be parameterized by the following formula

$$m_{\nu_\tau} = \left(10^{+3.5}_{-2.2}\right) \text{ eV} \left(\frac{10^{12} \text{ GeV}}{M_R}\right) \quad (16)$$

which holds with a 5% of accuracy in the range $M_R = (10^{11} - 10^{13})$ GeV, and 15% for $M_R = (10^{10} - 10^{14})$ GeV. Previous estimations of m_{ν_τ} in SUSY GUTs [15] correspond to M_R of the order M_G . The result (16) implies the $b - \tau$ unification, and the updated mass range of m_t was used.

The solution for α_{ν_τ} can be found explicitly in the approximation we used to derive the eq. (14). Moreover, since the eq. (8) does not contain α_b the approximate solution for α_{ν_τ} is reliable even for large values of $\tan\beta$. The neutrino mass can be represented in terms of $m_t(M_Z)$ as:

$$m_{\nu_\tau} = f_G f_Y \frac{m_t^2}{M_R} \quad (17)$$

where $f_G \equiv E_\nu(T_Z)/E_u(T_Z)$ describes the effect of gauge couplings renormalization, being $f_G \approx 0.145 \mp 0.01$ for $\alpha_s(M_Z) = 0.120 \pm 0.005$. The factor $f_Y = D_t(T_Z)^{1/2}/D_{\nu_\tau}(T_Z)$ represents the effect of the top and neutrino Yukawa renormalization. Confronting eq. (17) and (16) one finds that the Yukawa renormalization f_Y amounts to a factor larger than two[†]. For largest m_t allowed in the model we find $f_Y \sim 3$ at $M_R = 10^{-4}M_G$; f_Y can be as large as ~ 8 for a decoupled neutrino, $M_R = M_G$.

As in the case of the eq. (14) the Yukawa corrections depend only on $\alpha_t(0)$.

Consider now the effect of flavour mixing. Obviously, the results are not changed if the mixing is small both in the Dirac and the Majorana mass matrices. In the case of large mixing in the Yukawa matrices, it is necessary to consider the evolution of the full matrix system. Defining the 3×3 hermitian matrices α_x :

$$\alpha_x = \frac{Y_x \cdot Y_x^\dagger}{4\pi} \quad (x = u, d, e, \nu) \quad (18)$$

one can obtain the RGEs by the method of [10]. For example, for the matrix of the neutrino couplings one has

$$\alpha'_\nu = \sum_i b_\nu^i \alpha_i \alpha_\nu - \left(3\alpha_\nu^2 + \frac{1}{2}\{\alpha_\nu, \alpha_e\} + \text{Tr}(3\alpha_u + \alpha_\nu)\alpha_\nu\right) \quad (19)$$

[†]For solutions corresponding to low value of $\tan\beta$ (see fig. 3) the Yukawa contributions is approximately doubled.

Eq. (19) is valid above the mass scale of the heavier RH neutrino. In the case of large mixing the neutrino renormalization effect has smaller influence on the third family evolution, “discharging” partly on the light families evolution. However, the estimations show that this effect can not ensure the unification of the lighter down quarks and charged leptons, being of the order of 15% only.

Another deviation from the one family case can be related to large mixing in the Majorana sector. Consider two heaviest generations with Dirac mass matrix of the Fritzsch type, and the off-diagonal Majorana mass matrix \hat{M}_R . In this case one gets in lowest order:

$$m_{\nu\tau} = 2 \frac{m_{33}m_{23}}{M_R} \quad (20)$$

(and $\tan \theta \sim m_{23}/m_{33}$), where m_{ij} are the elements of the Dirac mass matrix. According to (20) the light mass is suppressed by a factor m_{23}/m_{33} in comparison with the case of diagonal matrix \hat{M}_R . The scale M_R should be m_{33}/m_{23} times smaller to get the same value of the light neutrino mass. Evidently, with diminishing M_R the renormalization effects due to neutrino increase.

5. Discussion and conclusions

1. In the minimally extended supersymmetric standard model we have found the relation between m_b/m_τ , α_s , $\tan \beta$, m_t and M_R . For masses M_R being 5 – 6 orders of magnitude smaller than M_G the renormalization effect due to the neutrino Yukawa couplings results in an increase of the predicted value of m_b/m_τ up to 10 – 15%. In a sense the neutrino renormalization compensates the effect of top quark interactions, which make it possible the $b-\tau$ unification at small $\tan \beta$. Neutrino renormalization effects disfavour the m_b-m_τ unification at small $\tan \beta$.

For fixed values of α_s and M_b , the $b-\tau$ unification gives the lower bounds on $\tan \beta$ and M_R (fig. 4). The smaller M_R , the larger $\tan \beta$. At M_R in the intermediate scale one gets $\tan \beta \gtrsim 43$ for $\alpha_s > 0.115$ and $M_b \leq 5.2$ GeV. Note that large values of $\tan \beta$ are especially interesting from the point of view of the unification of all Yukawa couplings as well as possible relations of the Yukawa and the gauge coupling.

2. The bounds on $\tan \beta$ and M_R are very sensitive to values of α_s and M_b used as an input. The increase of α_s up to the value, *e.g.* 0.125 (which is suggested by LEP data [16] and the unification of gauge couplings) increase the lower bound on $\tan \beta$ up to $\tan \beta \gtrsim 56$ for $M_b^{pole} \leq 5.2$ GeV. We used this conservative bound on M_b^{pole} which, in a sense, includes possible threshold effects (evaluated at $\sim 10\%$). The value $M_b^{pole} = 4.7 \pm 0.05$ GeV implied by [11] gives very strong bounds on $\tan \beta$ and M_R even for small values of α_s . Evidently,

future improvements of the determination of α_s and of M_b will allow to strengthen the conclusion.

3. According to the see-saw mechanism M_R determines masses of the light neutrinos (or give an upper bound on these masses if the matrix \hat{M}_R is appreciably non diagonal). Future measurements of the tau neutrino mass will allow to confirm the possibility of M_R being at the intermediate scale. In particular, an observation of the oscillation effects in the experiments CHORUS and NOMAD [17] will strongly suggest the ν_τ -mass in the cosmologically interesting domain. Also the confirmation of the MSW solution of the solar neutrino problem will testify for M_R in the $10^{10} - 10^{12}$ GeV range.
4. The violation of the bounds on $\tan\beta$ and M_R may testify for the existence of new particles below the GU scale, some other mechanism of neutrino mass generation, or the absence of the $b - \tau$, or $t - \nu_\tau$ unification.

In conclusion, the existence of the RH neutrinos at the intermediate scale (which is implied in particular by the see-saw mechanism and the existing hints from studies of solar, atmospheric and relic neutrinos) disfavours the $b - \tau$ unification, especially at low $\tan\beta$. The $b - \tau$ unification gives the lower bounds on M_R and $\tan\beta$. The bound can be strengthened by further refining the determination of α_s and of M_b .

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